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Numerical integration method of radiative exchange (NIMREX)

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Abstract—A method for calculating thermal radiation using quite a new concept, the NIMREX (numerical integration method of radiative exchange) model, is presented. The governing differential equations of thermal radiation are integrated by a method of numerical integration known from finite element methods. Theoretically the model is exact and can be brought to an arbitrary level of accuracy for an optical thickness from zero to infinity. For test purposes a FORTRAN code has been constructed. Based on a 1% accuracy level the NIMREX model is faster than a Monte Carlo zone (MCZ) method, except for the optically thin case. The two methods agree except in the optically thick case where the NIMREX model agrees with exact solutions of the diffusion method, while the MCZ method disagrees.

1. INTRODUCTION

THE THERMAL, stationary energy equation for a moving continuum with temperature as the dependent variable has the following general form [1]:

$$C(T) = D(T)+S(T) \tag{1}$$

where $C(T)$ is the convection term, $D(T)$ the diffusion term, and $S(T)$ the source term. The source term may arise from heat generation by burning of fuel, and/or may be a radiant source from the radiative exchange in the enclosure. Usually differential equation (1) is nonlinear and must be numerically solved by an iterative procedure.

The paper presents a new model, the NIMREX (numerical integration method of radiative exchange) model, for calculating the radiant source term. The hitherto known models of thermal radiation all have disadvantages. The most exact [2] of the known models are the zone method first presented by Hottel and Cohen [3] or the Monte Carlo methods [4]. For comparison with the present new model only analytic solutions and zone methods will therefore be used. The development of large and fast computers has brought the calculation cost and time of zone methods (including Monte Carlo methods) to a reasonable level. This means, that new models will be competitive to these, only if they have the same or higher accuracy at the same computer cost.

Two earlier almost identical models by Wu *et al.* [5] and Fernandes *et al.* [6] have generally speaking the same governing equations as the NIMREX model. The way of solving the equations is however quite different. Wu *et al.* and Fernandes *et al.* use a Galerkin finite element method (FEM) with numerical integration. The NIMREX model is not a finite element approach, but it uses the integration technique from FEM. The FEM models need a storage capacity which is proportional to the square of the nodal number. The NIMREX model solves the equations iteratively; the necessary storage capacity is only proportional to the nodal number and it has therefore no storage capacity limitations.

One important point in the understanding of the NIMREX model is the comprehension of the fact, that knowing the radiant source term at discrete *points* has the same value as knowing the mean radiant source term in discrete *zones* of the enclosure. In the zone method the enclosure is divided into a finite number of zones (or elements), and the radiant source term is calculated as a mean value of each zone. In the NIMREX model the enclosure (and the walls) are also divided into a number of elements, but the source term is calculated in the corner points of each element.

1.1. *Assumptions of the NIMREX model*

(1) The medium in the enclosure must be treated as a weighted sum of grey media [7] and/or dust clouds with known particular size distribution [8], each with separate temperatures and/or absorption coefficients a (and scattering coefficients s). Emission and scattering from the medium in the enclosure must be isotropic.

(2) The emissivity ε and the absorptivity α of the walls must be constant and independent of the wavelength of the radiation within each bandwidth of the grey media and each particle size. The emission and reflection from the walls must be Lambert diffuse.

(3) The variation of the extinction coefficient must be almost linear between two arbitrary chosen points in the enclosure with a mutual optical distance less than 3. The optical distance is equal to $k \cdot r$.

Two important limitations of the NIMREX model are the assumptions of isotropic radiation characteristics and almost linear variation of the extinction

NOMENCLATURE

a	absorption coefficient [m^{-1}]
A	area [m^2]
b_1, b_2, b_3	local coordinates of isoparametric element
dA_i	differential area of element i [m^2]
dq_{ij}^{in}	differential volumetric influx at point i from element j, $d^2Q_{j\to i}/dV_i$ [$W\,m^{-3}$]
$d^2Q_{j\to i}$	thermal power from element j into element i [W]
dV_i	differential volume of element i [m^3]
f	function
g	function
H	thermal influx at area element or point [$W\,m^{-2}$]
i	point or element
I	integration
j	point or element
$\bar{J}$	Jacobian matrix
k	extinction coefficient [m^{-1}]
$\bar{k}$	mean value of k between i and j [m^{-1}]
L	cube edge, distance between plates [m]
M	number of elements
n	integration order in numerical integration
q	volumetric thermal flux at a point [$W\,m^{-3}$]
r	distance between points i and j [m]
s	scattering coefficient [m^{-1}]
S	heat source term [$W\,m^{-3}$]
T	temperature [K]
V	volume [m^3]
W_i, W_j, W_m	weights to be used in numerical integrations
x, y, z	global coordinates [m].

Greek symbols

α	absorptivity of walls
β_j	angle between the normal vector at a wall point j and the direction vector to another point i
ε	emissivity of walls
$\theta(x)$	function given in ref. [13]
σ	Stefan–Boltzmann constant, 5.67×10^{-8} $W\,m^{-2}\,K^{-4}$.

Subscripts

area	area elements
i	element or point number i
j	element or point number j
vol	volume elements.

Superscripts

abs	absorbed part
in	total in
out	total out.

coefficient. In regions where large discontinuities of the extinction coefficient exist the NIMREX model would give incorrect results. As the NIMREX model is meant for engineering purposes these limitations are however not very restrictive, and very few engineering models of thermal radiation are less restrictive. The Monte Carlo zone (MCZ) method is less restrictive but it has, as will be shown later in this paper, some inaccuracy for optically thick zones. The discrete-ordinates method [9] is able to handle non-isotropic radiation, but it has some problems with so-called 'ray effects'. The ray effects are especially pronounced if there are local radiation sources in the medium or if absorption has an appreciable importance compared to scattering.

2. GOVERNING EQUATIONS

2.1. *Radiative exchange between infinitesimal elements*

In the formulation of the governing equations for thermal radiation it is useful to look at infinitesimal elements in an enclosure (Fig. 1). Both volume and area elements are considered.

In writing the thermal radiative exchange between infinitesimal elements four factors should be multiplied (Table 1). An emitter-element j and a receiver-element i are considered.

(1) The first factor is the total thermal power emitted from the emitter.

(2) The second factor is the direct view factor from the emitter to the receiver, i.e. the part of the radiation from j that could be received by i without extinction between the two elements.

(3) The third factor is the part of the radiation from j to i which is not absorbed or scattered on its way, i.e. the transmitted part.

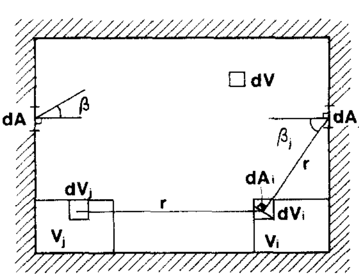

FIG. 1. Two-dimensional illustration of elemental division of an enclosure.

Table 1. Governing factors for radiative exchange between infinitesimal elements

Emitter	Receiver	Total thermal power from emitter, dQ_j^{out} [W]	F_{ji}	Transmitted	Absorbed or scattered by receiver	Thermal power from emitter into receiver, $d^2Q_{j\to i}$ [W]
dA_j	dA_i	$dA_j[\varepsilon\sigma T_j^4+(1-\alpha)H_j]$	$\frac{\cos\beta_j}{\pi}\frac{\cos\beta_i\,dA_i}{r^2}$	$e^{-\bar{k}r}$	1	$\frac{e^{-\bar{k}r}}{\pi r^2}\cos\beta_i\,dA_i\cos\beta_j\,dA_j[\varepsilon\sigma T_j^4+(1-\alpha)H_j]$
dA_j	dV_i	$dA_j[\varepsilon\sigma T_j^4+(1-\alpha)H_j]$	$\frac{\cos\beta_j}{\pi}\frac{dA_i}{r^2}$	$e^{-\bar{k}r}$	$k_i\frac{dV_i}{dA_i}$	$\frac{e^{-\bar{k}r}}{\pi r^2}k_i\,dV_i\cos\beta_j\,dA_j[\varepsilon\sigma T_j^4+(1-\alpha)H_j]$
dV_j	dA_i	$dV_j\left[4a_j\sigma T_j^4+q_j^{in}\frac{s_j}{k_j}\right]$	$\frac{1}{4\pi}\frac{\cos\beta_i\,dA_i}{r^2}$	$e^{-\bar{k}r}$	1	$\frac{e^{-\bar{k}r}}{\pi r^2}\cos\beta_i\,dA_i\,dV_j\left[a_j\sigma T_j^4+\frac{1}{4}q_j^{in}\frac{s_j}{k_j}\right]$
dV_j	dV_i	$dV_j\left[4a_j\sigma T_j^4+q_j^{in}\frac{s_j}{k_j}\right]$	$\frac{1}{4\pi}\frac{dA_i}{r^2}$	$e^{-\bar{k}r}$	$k_i\frac{dV_i}{dA_i}$	$\frac{e^{-\bar{k}r}}{\pi r^2}k_i\,dV_i\,dV_j\left[a_j\sigma T_j^4+\frac{1}{4}q_j^{in}\frac{s_j}{k_j}\right]$

(4) The fourth factor gives the part of the radiation into i, which is absorbed or scattered.

By multiplying these four factors the total radiative power from element j, that is received by element i, is obtained. This term appears in the last column in Table 1. The term dA_i for the volume element dV_i is the cross-sectional area of dV_i looking from element j. In the multiplication this term disappears. The term $\bar{k}$ is the mean value of the extinction coefficient between elements i and j. In this model it is assumed that $\bar{k}$ can be approximated by

$$\bar{k} = (k_i+k_j)/2. \qquad (2)$$

This is the reason for assumption (3) in Section 1.1. If the variation of k is linear between i and j, then equation (2) is exact [10].

2.2. *Integration of the expressions*

In Hottel and Cohen's zone method the above-mentioned terms are integrated numerically by a double volume integration. In the Monte Carlo method they are integrated statistically. With a sufficiently large number of zones, these two methods are exact and, of course, give the same results. In an earlier model by Rasmussen [10] the integrations were carried out by empirical methods, which gave approximate results.

A double volume integration is, however, not necessary. Taking, for example, the volume to volume expression, assuming no scattering, i.e. $a_j = k_j$ and $s_j = 0$, the following expression for the volumetric influx at point i (infinitesimal element) from element j can be written. The expression occurs by division with dV_i

$$dq_{ij}^{in} = k_i\frac{e^{-\bar{k}r}}{\pi r^2}k_j\sigma T_j^4\,dV_j. \qquad (3)$$

This expression is integrated across an element V_j of finite size

$$q_{ij}^{in} = \frac{k_i\sigma}{\pi}\int_{V_j}\frac{e^{-\bar{k}r}}{r^2}k_jT_j^4\,dV_j. \qquad (4)$$

Element V_j of finite size appears as the element between six planes. Orthogonality is not necessary. Each volume V_j has eight corner points, in which the radiant source term is calculated. By summing up equation (4) over the total number of volume elements in the enclosure the total radiant *influx* at point i from the entire enclosure occurs. In this example, only volumetric exchange is considered

$$q_i^{in} = \frac{k_i\sigma}{\pi}\sum_{j=1}^{M_{vol}}\int_{V_j}\frac{e^{-\bar{k}r}}{r^2}k_jT_j^4\,dV_j \qquad (5)$$

where M_{vol} is the total number of volume elements. The volumetric radiant *outflux* from element i is given by

$$q_i^{\text{out}} = 4k_i\sigma T_i^4. \quad (6)$$

Finally the radiant source term at point i appears as

$$q_i = q_i^{\text{in}} - q_i^{\text{out}} = \frac{k_i\sigma}{\pi}\sum_{j=1}^{M_{\text{vol}}}\int_{V_j}\frac{e^{-\bar{k}r}}{r^2}k_jT_j^4\,dV_j - 4k_i\sigma T_i^4. \quad (7)$$

The integration and summation of equation (7) is carried out for every discrete point (cornerpoints of the elements) in the enclosure. By this procedure the radiant source term field is found. It is represented by the value of the source term in each of the discrete points. This source term is then put in as a part of the source term $S(T)$ in equation (1), which is solved iteratively.

If pure radiation is considered, and radiant heat exchange is the only heat exchange, then equation (1) is reduced to $q_i = 0$ or

$$q_i^{\text{in}} = q_i^{\text{out}}. \quad (8)$$

This equation is then, like equation (1), solved iteratively.

The same procedure, as above, is used in the calculation of the influx on area elements and in calculations, where scattering and reflection is included. When the calculations are carried out for volume elements as well as for area elements, and with scattering and reflection included, the following equations for the total area influx and net volumetric influx occur:

$$H_i = \sum_{j=1}^{M_{\text{area}}}\int_{A_j}\frac{e^{-\bar{k}r}}{\pi r^2}\cos\beta_i\cos\beta_j[\varepsilon\sigma T_j^4 + (1-\alpha)H_j]\,dA_j + \sum_{j=1}^{M_{\text{vol}}}\int_{V_j}\frac{e^{-\bar{k}r}}{\pi r^2}\cos\beta_i\left[a_j\sigma T_j^4 + \frac{1}{4}q_j^{\text{abs}}\frac{s_j}{a_j}\right]dV_j \quad (9)$$

$$q_i^{\text{abs}} = \frac{a_i}{\pi}\left[\sum_{j=1}^{M_{\text{area}}}\int_{A_j}\frac{e^{-\bar{k}r}}{r^2}\cos\beta_j(\varepsilon\sigma T_j^4 + (1-\alpha)H_j)\,dA_j + \sum_{j=1}^{M_{\text{vol}}}\int_{V_j}\frac{e^{-\bar{k}r}}{r^2}\left(a_j\sigma T_j^4 + \frac{1}{4}q_j^{\text{abs}}\frac{s_j}{a_j}\right)dV_j\right]. \quad (10)$$

In the derivation of equations (9) and (10) the following definition is used:

$$q_i^{\text{abs}} = q_i^{\text{in}}\frac{a_i}{k_i}. \quad (11)$$

To find the volumetric source term of point i the following equation is used:

$$q_i = q_i^{\text{abs}} - 4a_i\sigma T_i^4. \quad (12)$$

The net radiant flux to an area element is given by

$$H_i^{\text{net}} = \alpha H_i - \varepsilon\sigma T_i^4. \quad (13)$$

These equations conclude the derivation of the expressions giving the volumetric radiant source terms at internal points and the radiant area flux to walls of an enclosure with radiant heat exchange. They appear as a summation of single volumetric or area integrations. The next section shows the numerical integration of these equations.

3. NUMERICAL INTEGRATION

The first step of the numerical integration is the transformation of the element (volume or area) for integration into a so-called isoparametric element. This element has in each direction the boundaries at -1 and 1. The global coordinates x, y and z are transformed into the local isoparametric coordinates b_1, b_2 and b_3. The functions $x(b_1, b_2, b_3)$, $y(b_1, b_2, b_3)$ and $z(b_1, b_2, b_3)$ are known. The transformation and the advantages of this is well described by Zienkiewicz [11].

Take for example the function f to be integrated in the volume element V_j. The integration is carried out as follows:

$$I = \int_{V_j} f(x,y,z)\,dx\,dy\,dz = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} g(b_1,b_2,b_3)\,db_1\,db_2\,db_3 \quad (14)$$

with

$$g = f(x,y,z)|\det\bar{\bar{J}}|.$$

Matrix $\bar{\bar{J}}$ is the Jacobian matrix of the transformation [11, 12].

By the use of Gauss–Legendre quadrature the integration of equation (14) is then carried out numerically as follows [11]:

$$I = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\sum_{m=1}^{n_3} W_iW_jW_m g(b_{1i}, b_{2j}, b_{3m}). \quad (15)$$

The terms n_1, n_2 and n_3 are the order of integration in each of the three directions in the isoparametric element. (Usually $n_1 = n_2 = n_3$.) The terms b_{1i}, b_{2j} and b_{3m} are the local coordinates of the integration points in the element, and W_i, W_j, W_m are the weights to be used in the summation. If the order of integration is n, then a polynomial of degree $2n-1$ can be integrated exactly by this method. Tables of weights and coordinates of integration points for different orders of integration can be found in ref. [11].

The next step is the introduction of this method in the integration of equations like equations (9) and (10). Because of the exponential function the order of integration should be rather high, when the exponent has an appreciable value. Terms which are known only in the corner points of the elements have to be interpolated within the elements. Linear interpolation is used for the following terms: k, s, a, α, ε, T^4 or T, H and q. In the computer code developed for the test of the NIMREX model the order of integration is between 1 and 6, depending on the function to be integrated. For elemental optical thickness between 0 and 4 the code gives the radiant source terms within an accuracy of $\pm 1\%$. All percentages are related to net outflux from the point, i.e. $4a_i\sigma T_i^4$. (Only volumetric source terms have been considered.) The upper limit of optical thickness and the level of accuracy are arbitrarily chosen and can both be increased, if required. This, of course, extends computer calculation time.

4. EXAMPLES TO TEST THE NIMREX MODEL

In this section results of examples used to test the NIMREX model are shown. In the first two examples exact solutions are known either from analytic solutions or from the MCZ method. The third example, in which NIMREX calculations are not included, shows how the zone method is inaccurate in the optically thick case. Only pure radiation examples have been considered. As stated by Wu *et al.* [5] such examples are the most sensitive to the accuracy of the radiation models.

4.1. *Calculation of a cubic enclosure*

This example concerns a cubic enclosure with black walls, i.e. no reflection at the walls. The cube is a unity-cube with the reference point at one corner. The temperature of the walls is 0 K. In the enclosure there is no scattering and the extinction coefficient is constant. The temperature field is given by one of the following equations:

$$T = T_0(1+15xyz)^{0.25} \tag{16}$$

or

$$T = T_0(1+xyz) \tag{17}$$

where T_0 is a constant. The reason for the chosen temperature field of equation (16) is the solution in the optically thick case. In this case, the diffusion method of thermal radiation is valid [7]. The radiant source term is then given by

$$q_i = \frac{4\sigma}{3k}\left[\frac{\partial^2 T^4}{\partial x^2}+\frac{\partial^2 T^4}{\partial y^2}+\frac{\partial^2 T^4}{\partial z^2}\right]. \tag{18}$$

By the temperature field from equation (16) this source term must be zero. When this temperature field is used, the term T^4 is linearly interpolated within the elements. To get a reasonable variation of the temperature in the calculation domain the constant 15 is chosen in equation (16). This gives a factor 2 between the maximum and minimum temperature. In the optically thick case the NIMREX model was compared with the solution of equation (18). In the optically thin case equation (17) was used and the NIMREX model was compared with Monte Carlo solutions of the zone method. In this case the term T was linearly interpolated. In all calculations the cubic enclosure was divided into $5\times5\times5$ zones. In calculations with the NIMREX model the radiant source term was found at each discrete point of the enclosure, and for the MCZ method as a mean value in each zone.

In the optically thick case the NIMREX model agreed with the analytic solutions within an accuracy of $\pm1\%$. The upper limit of elemental optical thickness was 4 and the integration order was 6. For lower optical thickness, the integration order was decreased. The limits for the different integration orders were found by comparing a higher order solution with a lower order solution for a given optical thickness. When the difference between the results was small, say 1%, the higher order solution was considered as effectively exact, and the difference was the error of the lower order.

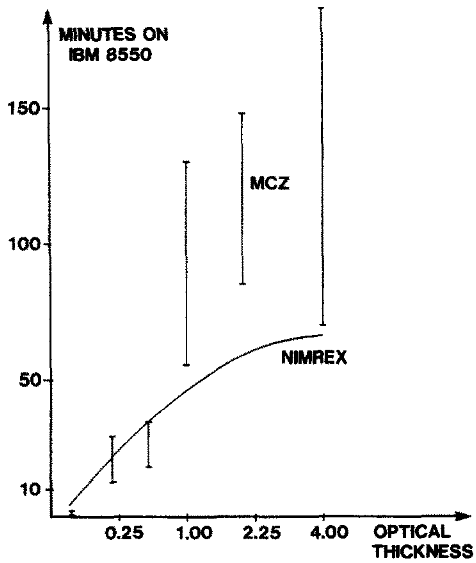

FIG. 2. Computing time for the MCZ and NIMREX methods on an IBM 8550 (1% accuracy for both methods).

In the optically thin case the NIMREX model agreed very well with the MCZ method within the accuracy of the two methods. The computer used for the calculations was an IBM 8550. The MCZ method used in this work is given in the Appendix. Figure 2 shows the time consumption of the two methods both brought to an accuracy of $\pm1\%$. In the MCZ method the necessary computing time is inversely proportional to the square of the uncertainty. In the present calculations the necessary computing time is therefore found by this relation from the actual values of computing time and accuracy. The actual computing time was the same as with the NIMREX model. The accuracy of the MCZ method is a statistical quantity and has in itself some uncertainty. For this reason the calculation time for a given accuracy is uncertain. In Fig. 2 the calculation time of the MCZ method is therefore shown as bars instead of a curve. The top point on the bars is found from the maximum uncertainty and the bottom point from the mean uncertainty in the calculation domain. The figure shows, that for elemental optical thickness below 0.5 the MCZ method is slightly faster than the NIMREX model. Above this limit the NIMREX model is faster. In the optically thick case, i.e. optical thickness above 1, the

Table 2. The dependence on the C_is of the optical distance between the plates: $\theta(x) = C_1 - C_2(x-0.5)^2 - C_3(x-0.5)^4$

kL	C_1	C_2	C_3
0.1	0.30	0.01222	0.1111
0.5	0.47	0.1897	0.3611
1.0	0.70	0.4631	1.0278
2.0	1.28	1.5853	2.1389

two methods disagreed slightly (1%), which gives an indication of the inaccuracy of the MCZ method.

Another test was carried out by an asymmetric zonal division of the enclosure. The temperature field was symmetric, and of course the results of the calculations should be symmetric too. The enclosure was divided into $4 \times 8 \times 4$ zones. The asymmetry of the calculated radiant source terms with the NIMREX model was in all cases less than 0.1%. On the other hand the MCZ method gave asymmetric results of up to 2% in the optically thick case. This example shows, that when the linear interpolation of the terms within the element is exact, then the NIMREX model is independent of the zonal division.

4.2. *Heat source term between two infinite plates*

This example concerns the temperature field between two infinite plates with a mutual distance of 1. The temperature of the two plates is in this case 0 K, and the reflection is zero. The medium between the plates has no scattering, and the extinction coefficient k is constant. A constant heat source S is introduced between the plates.

Heaslet and Warming [13] have solved this problem analytically. The solution is

$$T(x)^4 = \theta(x)\frac{S}{k\sigma} \quad (19)$$

where $\theta(x)$ is a function shown in a figure in ref. [13]. x is the local distance from one of the plates.

For the purpose of the present calculations curves were fitted to the function $\theta(x)$ taken from this figure of ref. [13], and from these curves the temperature field was found. The curve-fitted expression of $\theta(x)$ has the following form:

$$\theta(x) = C_1 - C_2(x-0.5)^2 - C_3(x-0.5)^4. \quad (20)$$

Table 2 shows the C_is for different optical distances $k \cdot L$ between the plates.

The region between the plates was divided into four zones across this region. The result of the calculations was the heat balance between the received and the emitted radiation of each discrete point. The calculations were carried out for different optical distances between the plates. For the optical distances of 2.0, 1.0, 0.5 and 0.1 the errors in the heat balance were less then 4.0, 3.0, 2.5 and 2.5%, respectively. These errors *include* errors of the curve-fitting. All percentages are as in the previous example related to the term $4a_i\sigma T_i^4$. For an optical distance of 2.0 the number of zones between the plates was increased to 10. By this operation the errors were halved.

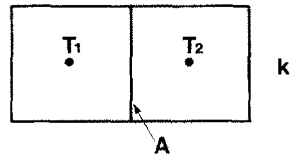

FIG. 3. Two cubes with a common plane.

The results of the calculations in this example show that the NIMREX model is almost exact, and that the errors are decreased with increasing number of zones. The reason for the latter is that the linear interpolation of T^4 used here is not exact (but indeed better than the linear interpolation of T).

4.3. *Inaccuracy of the zone method*

This example will show (at least qualitatively) a case, where the zone method is inaccurate in the optically thick region. Imagine a region inside a furnace with a linear variation of temperature in that region. The extinction coefficient is constant and there is no scattering. Two adjoining cubic zones are placed in this region. The length of the cube edge is L (Fig. 3).

The temperature difference $\Delta T = T_2 - T_1$ is small compared to the absolute temperature. The radiant heat flux across area A is calculated, and normalized by the term $A\sigma(T_2^4 - T_1^4)$. The correct value of this heat flux has the following variation. In the optically thin case the heat flux is increased with increasing extinction coefficient. A maximum is reached, and in the optically thick case the heat flux decreases with increasing extinction. Figure 4 shows the results of calculations carried out by different methods. The NIMREX model is not included on this figure, because it is not possible directly to calculate the area flux term with the NIMREX model.

In the optically thick case, i.e. optical thickness greater than about 4, the diffusion method is correct and gives decreasing heat flux with increasing optical thickness. The MCZ method, however, approaches 1.0

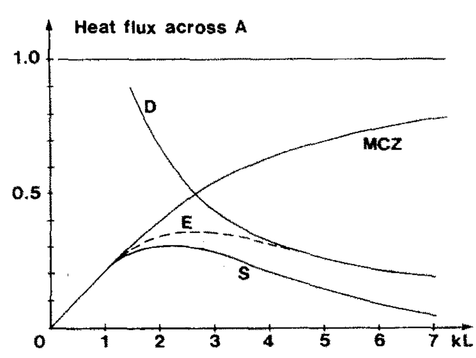

FIG. 4. Heat flux between two cubic zones. No scattering. MCZ, Monte Carlo zone method [14]; D, diffusion method [7]; S, simplified method [10]; E, asymptotically exact for small and large optical thickness.

with increasing optical thickness. The MCZ method gives in this case the same result as the heat flux between two plates of area A, quite close and with the temperatures T_2 and T_1. This result is obviously wrong.

In the optically thin case the diffusion method fails, and the MCZ method gives correct results. The curve marked S is the result of calculations carried out with an earlier model by Rasmussen [10]. This model has the right shape, but fails in optically thick cases. Curve E shows qualitatively the correct shape. This curve indicates that the MCZ method begins to fail for an optical thickness of about 1, and that the diffusion method is correct from about 4. The reason for the failure of the MCZ method is the double volume integration and the assumption of constant temperature across each zone.

When calculating the radiant source term, however, the error of the MCZ method is in this example an order of magnitude smaller. This is due to the fact that the source term, or the volumetric radiant flux term, is the first derivative of the area flux term calculated above. In other words, the error on one side of a zone element is almost fully balanced with the error on the opposite side.

5. CONCLUSIONS

In the NIMREX model the well-known and well-established method of numerical integration in finite element methods and the theoretical governing equations of thermal radiation with isotropic characteristics have been brought together. The result is an excellent tool of numerical calculation in thermal radiation for engineering purposes. Theoretically the NIMREX model covers optical thickness from zero to infinity.

From the calculated examples and from the theory it has been shown that the NIMREX model is exact and can be brought to an arbitrary level of accuracy, dependent on the discretization (elemental number) and order of numerical integration.

Based on a 1% accuracy level the NIMREX model is faster than an MCZ method, except for the optically thin case. With the assumptions mentioned in the beginning of this paper being fulfilled, the NIMREX model is considered competitive with the best of the well-known numerical methods of radiative heat exchange.

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APPENDIX: DESCRIPTION OF MONTE CARLO METHOD

In a Monte Carlo calculation the radiation is simulated by tracing a number of discrete energy bundles or rays. Siegel and Howell [4] have described the principle in detail. The code applied in this work is MOCARAD [14]; a general code for predicting radiative heat transfer in an absorbing/emitting enclosure. Since Monte Carlo merely is a name given to a whole family of methods, some specific features of MOCARAD as used in this work are given below.

The emitted energy from all zones is represented by a number of energy bundles all having the same energy. Thus, the zones emitting the most energy also emit the most energy bundles.

Points of emission are chosen in a semi-random fashion. Each zone is subdivided into a number of uniform cells, all of which emit the same number of energy bundles. Within each cell the points of emission are chosen randomly.

The directions of the rays are chosen in a deterministic way as proposed by Richter and Heap [15].

After emission the energy bundles are traced through the zones and exponentially lose energy according to the optical pathlength travelled in each zone. A 2% cut-off level of initial energy is used to stop tracing; the remaining energy is absorbed by the zone presently containing the energy bundle.

METHODE D'INTEGRATION NUMERIQUE DE L'ECHANGE RADIATIF (NIMREX)

Résumé—On présente une méthode de calcul du rayonnement thermique à partir d'un nouveau concept, le modèle NIMREX (intégration numérique de l'échange radiatif). Les équations différentielles fondamentales sont intégrées par une méthode numérique d'intégration dérivant des méthodes d'éléments. Théoriquement le modèle est exact et il peut être porté à un niveau arbitraire de précision pour une épaisseur optique allant de zéro à l'infini. Pour l'essayer, un code FORTRAN a été construit. Basé sur un niveau de précision de 1%, le modèle NIMREX est plus rapide que la méthode de zone Monte Carlo (MCZ), excepté pour le cas optiquement mince. Les deux méthodes s'accordent, excepté dans le cas optiquement épais où le modèle NIMREX s'accorde avec les solutions exactes de la méthode de diffusion, tandis que la méthode MCZ s'en différencie.

EIN NUMERISCHES VERFAHREN ZUR BERECHNUNG DES STRAHLUNGS-WÄRMEAUSTAUSCHES

Zusammenfassung—Es wird ein neuartiges Verfahren zur Berechnung des Wärmeaustausches durch Strahlung, das NIMREX-Modell, vorgestellt. Die maßgeblichen Differentialgleichungen der thermischen Strahlung werden durch ein numerisches Integrationsverfahren, welches von der Finite-Element-Methode her bekannt ist, gelöst. Das Modell ist theoretisch exakt und rechnet mit bliebiger Genauigkeit bei optischen Dicken zwischen null und unendlich. Für Testzwecke wurde ein Fortran-Program erstellt. Bei einer Rechengenauigkeit von 1 Prozent ist, mit Ausnahme des optisch dünnen Falles, das NIMREX-Modell schneller als ein Monte-Carlo-Zonen (MCZ)-Verfahren. Die beiden Methoden stimmen miteinander überein, mit Ausnahme des optisch dicken Falles, wo das NIMREX-Verfahren im Gegensatz zum MCZ-Verfahren mit den exakten Lösungen der Diffusions-Methode übereinstimmt.

МЕТОД ЧИСЛЕННОГО ИНТЕГРИРОВАНИЯ ЛУЧИСТОГО ПЕРЕНОСА

Аннотация—Представлен метод расчета теплового излучения на основе принципиально новой модели—метода численного интегрирования лучистого переноса (МЧИЛП). Определяющие дифференциальные уравнения теплового излучения интегрируются численно одним из методов конечных элементов. В теоретическом плане модель является точной и позволяет рассчитывать оптическую толщину с любой степенью точностиот нуля до бесконечности. С целью проверки модели составлена программа на языке ФОРТРАН. При 1% уровне точности модель МЧИЛП позволяет более быстро полчать результаты, чем зональный метод Монте Карло (ЗМК), за исключением случаев с малой оптической толщиной. Оба метода дают совпадающие результаты, кроме случаев с большой оптической толщиной, в которых модель МЧИЛП, в отличие от модели ЗМК, согласуется с точными решениями, полученными диффузионным методом.